Optimisation de forme pour la conception des cavités de liner

Gilles Tissot, Gwénaël Gabard

Shape optimisation for acoustic liners design

Gilles Tissot, Gwénaël Gabard

Laboratoire d'Acoustique de l'Université du Mans

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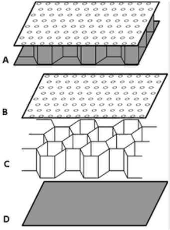
Context

Acoustic liners:



Schematic representation.

Integrated in nacelles of aricrafts.

Constraints:

- Low frequency absorption requires deep cavities.
- Challenge to push away this limitation.

MACIA ANR project: LAUM, SAFRAN.

Context

Objective:

- Optimise cavity shape of liners.
- Make it resonate at lower frequencies.

Strategy:

- Shape optimisation.
- Based on frequency response.

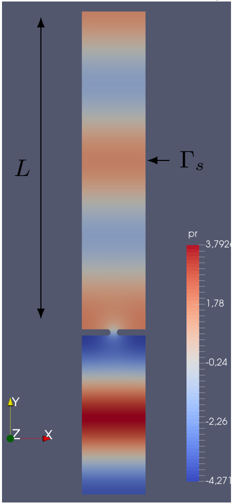
Outline

Governing equations

Bossart et al. (2003)
Berggren et al. (2018)

Helmholtz equation:

$$\begin{cases} \Delta p + k^2 p = 0 \quad \text{in } \Omega \\ \nabla p . \boldsymbol{n} = 0 \quad \text{on } \Gamma_s \textit{ (free slip)}. \end{cases}$$



Governing equations

Bossart et al. (2003)
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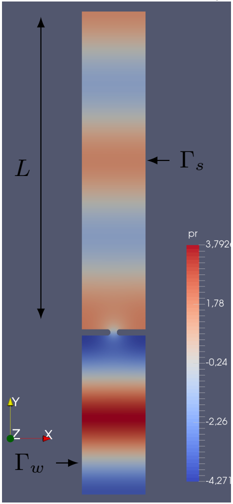
Helmholtz equation:

$$
\begin{cases}
\Delta p + k^2 p = 0 \quad \text{in } \Omega \\
\nabla p.\boldsymbol{n} = 0 \quad \text{on } \Gamma_s \ \textit{(free slip)} \\
\nabla p.\boldsymbol{n} = \delta_v \frac{i-1}{2} \Delta_T p - \delta_T k^2 \frac{(i-1)(\gamma-1)}{2} p \\
\qquad\qquad\qquad\qquad \text{on } \Gamma_w \ \textit{(walls)}.
\end{cases}
$$



- $\delta_v = \sqrt{\frac{2}{\omega Re_a}}$; $\delta_T = \sqrt{\frac{2}{\omega Re_a Pr}}$.
- $\nabla f = \nabla_T f + \frac{\partial f}{\partial n}\boldsymbol{n}$;

  $\Delta f = \Delta_T f + \frac{\partial^2 f}{\partial n^2} + \nabla_T.\boldsymbol{n}\frac{\partial f}{\partial n}$

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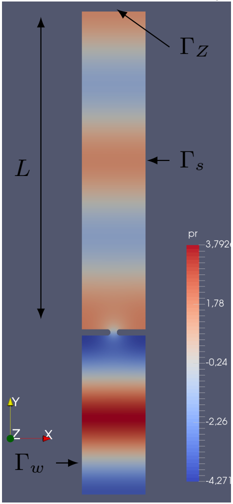

- Unitary amplitude of incoming plane wave.
- Reflexion coefficient:

$$R = \left(\frac{1}{L_Z} \int_{\Gamma_Z} p(x, L)\, \mathrm{d}x - e^{ikL} \right) e^{ikL}.$$

- Impedance: $Z = \dfrac{1+R}{1-R}$.

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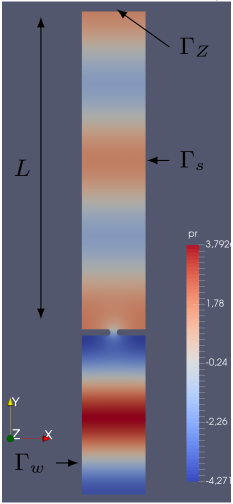


- Modify cavity shape for improving behaviour: *Target impedance.*

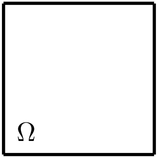
$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p,k) - Z_T(k)|^2 \, \mathrm{d}k$$

Shape optimisation

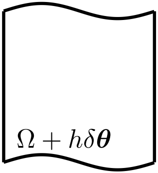
Generalities

Infinitesimal domain variations:

Reference domain:



Shape variation:



h small.

Shape optimisation

Generalities

Allaire (2003,2004)

Cost functional: unconstrained minimisation problem

$$\mathcal{J}(\Omega) = \int_{\Omega} f(x)\,\mathrm{d}x$$

Shape derivative: $\forall \delta\boldsymbol{\theta}$ we have

$$(\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}, \delta\boldsymbol{\theta}) = \lim_{h\to 0} \frac{\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) - \mathcal{J}(\Omega)}{h} = \int_{\Gamma_\theta} \delta\boldsymbol{\theta}(x).\boldsymbol{n}(x)\, f(x)\,\mathrm{d}x.$$

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Descent direction:

$$\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) + h\int_{\Gamma_\theta} \delta\boldsymbol{\theta}(x).\boldsymbol{n}(x)\, f(x)\,\mathrm{d}x + \mathcal{O}(h^2)$$

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$\Rightarrow \delta\boldsymbol{\theta}(x) = -f(x)\boldsymbol{n}(x)$ ensures a descent direction!

Indeed: $\mathcal{J}(\Omega + h\delta\boldsymbol{\theta}) = \mathcal{J}(\Omega) - h\int_{\Gamma_\theta} f^2(x)\,\mathrm{d}x + \mathcal{O}(h^2)$

Shape optimisation

Impedance matching

Bängtsson et al. (2003)

Cost functional:

$$\mathcal{J}(\Omega, p) = \frac{1}{2} \int_{k_1}^{k_2} |Z(p,k) - Z_T(k)|^2 \, \mathrm{d}k$$

subject to

$$\Delta p + k^2 p = 0 \quad + \quad \text{B.C.}$$

Impedance: $\quad Z(p,k) = \dfrac{e^{-ikL} - e^{ikL} + \bar{p}}{e^{-ikL} + e^{ikL} - \bar{p}} \quad$ with $\quad \bar{p} = \dfrac{1}{L_Z} \displaystyle\int_{\Gamma_Z} p \, \mathrm{d}l.$

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subject to

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We define the Lagrangian *(constrained $\mapsto$ unconstrained)*

$$\mathcal{L}(\Omega, p, \lambda) = \mathcal{J}(\Omega, p) - \text{real}\left(\int_{k_1}^{k_2} \left(\lambda, \Delta p + k^2 p\right)_\Omega \, \mathrm{d}k \right)$$

Shape optimisation

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Derivative with respect to each variable:

$(\nabla_{\delta\lambda}\mathcal{L}, \delta\lambda) = (\nabla_{\delta p}\mathcal{L}, \delta p) = 0$

$(\nabla_{\delta\boldsymbol{\theta}}\mathcal{L}, \delta\boldsymbol{\theta}) = 0 \Rightarrow (\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}, \delta\boldsymbol{\theta})$

Impedance matching

Derivative with respect to λ: $\Delta p + k^2 p = 0.$

Impedance matching

Derivative with respect to λ: $\Delta p + k^2 p = 0.$

Derivative with respect to p: Adjoint equation

$$\begin{cases} \Delta\lambda + (k^*)^2\lambda = 0 & x \in \Omega \\ \nabla\lambda.\boldsymbol{n} = 0 & x \in \Gamma_s \\ \nabla\lambda.\boldsymbol{n} + \delta_v \dfrac{i+1}{2}\Delta_T\lambda - \delta_T (k^*)^2 \dfrac{(i+1)(\gamma-1)}{2}\lambda = 0 & x \in \Gamma_w \\ \nabla\lambda.\boldsymbol{n} - ik^*\lambda = \dfrac{2}{L_Z} \dfrac{e^{ikL}(Z(p,k) - Z_T(k))}{\left((e^{-ikL} + e^{ikL} - \bar{p})^2\right)^*} & x \in \Gamma_Z. \end{cases}$$

Impedance matching

Derivative with respect to λ: $\Delta p + k^2 p = 0$.

Derivative with respect to p: Adjoint equation

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Derivative with respect to Ω: For any displacement direction $\delta\boldsymbol{\theta}$,

$$(\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}, \delta\boldsymbol{\theta}) = \int_{\Gamma_\theta} (\delta\boldsymbol{\theta}(x).\boldsymbol{n}(x)) \int_{k_1}^{k_2} \text{Real}\left[-\nabla\lambda^*.\nabla p + k^2\lambda^* p\right] \,\mathrm{d}k \,\mathrm{d}x.$$

Impedance matching

Algorithm:

Solve iteratively:

- Solve Helmholtz direct $\Rightarrow$ $p(k)$ *(frequency response)*.
- Solve Helmholtz adjoint $\Rightarrow$ $\lambda(k)$ *(frequency response)*.
- Optimality condition $\Rightarrow$ $\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}$.
- Move shape with linesearch algorithm *(Armijo backtracking)*.
- Iterate until convergence.

Numerical implementation

Moës et al. (1999)
Osher et al. (2001)
Protas et al. (2004)

Finite elements with immersed boundary:

- XFEM for integration methods on cut elements (GETFEM++).
- Level set:

$$\begin{cases} \psi = 0 & x \in \Gamma_\theta \\ \psi > 0 & x \in \Omega \\ \psi < 0 & x \notin \Omega. \end{cases}$$

- Transport equation for moving the shape:

$$\frac{\partial \psi}{\partial t} + v\boldsymbol{n}.\nabla\psi = 0.$$

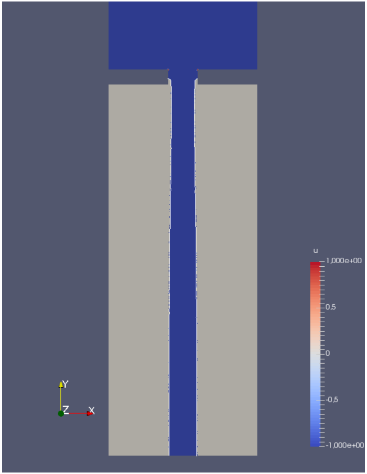
with

$$v = -\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}.\boldsymbol{n}. \quad \textit{(Regularised by Sobolev Gradient)}$$

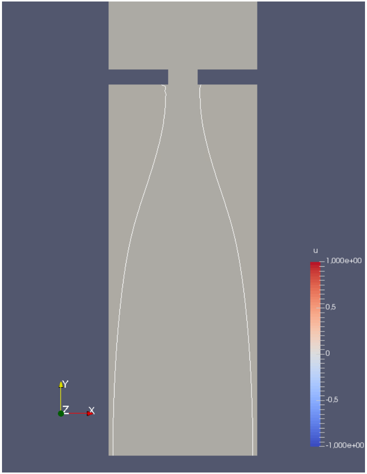
$\boldsymbol{n}$ is the outward wall normal direction. SUPG discretisation.

Results

No viscosity: $Z_T = 0$, $k = [0.2 : 0.6]$



Initial condition.



Converged.

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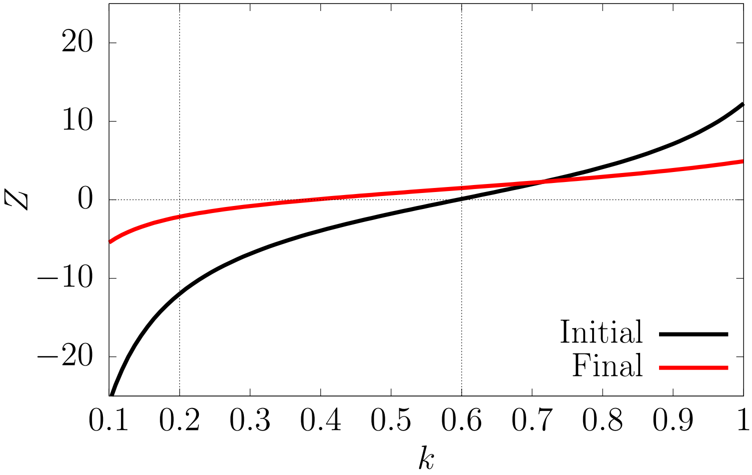
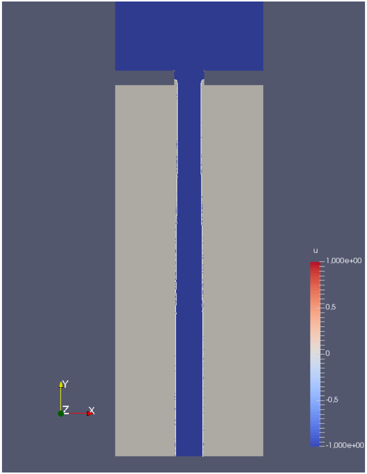


Reactance $Im(Z)$.

Results

No viscosity: $Z_T = 0,\ k = [0.15 : 0.25]$



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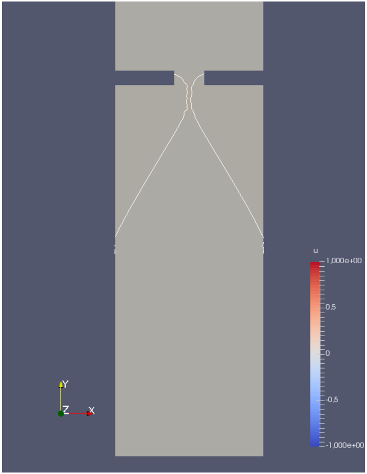

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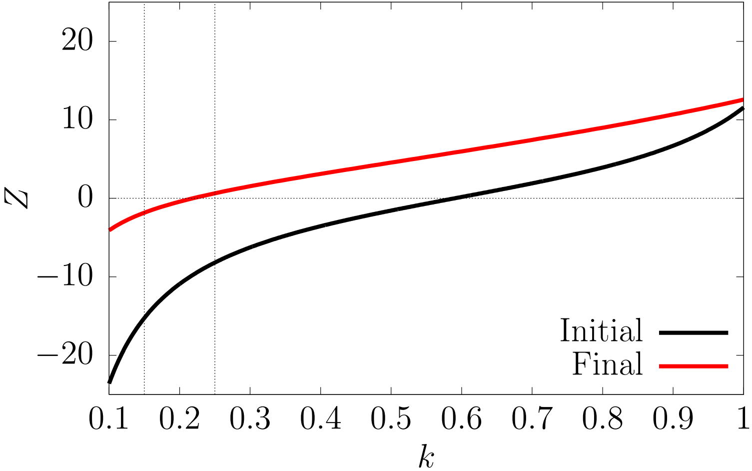
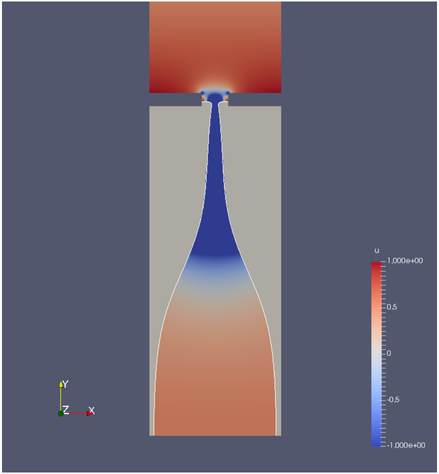


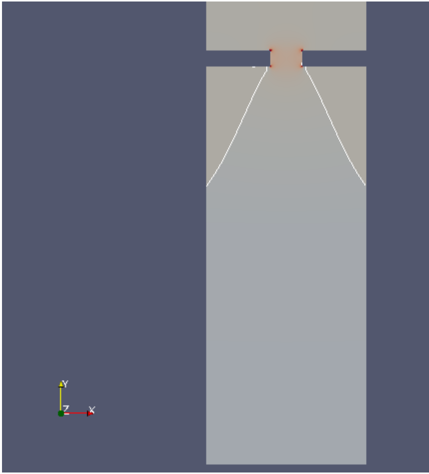
Reactance $Im(Z)$.

Results

Viscosity: $Z_T = 1.0$, $k = [0.2 : 0.6]$



Initial condition.
Coloured by $\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}$.



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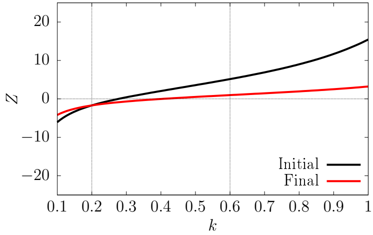
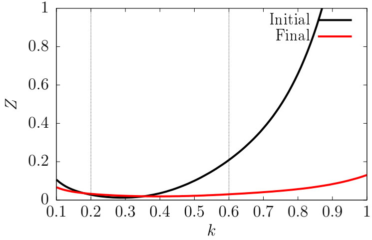
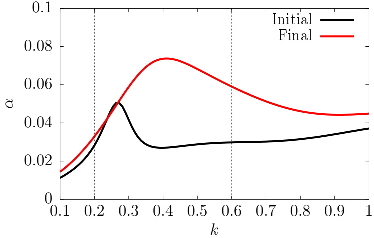


Reactance $Im(Z)$.



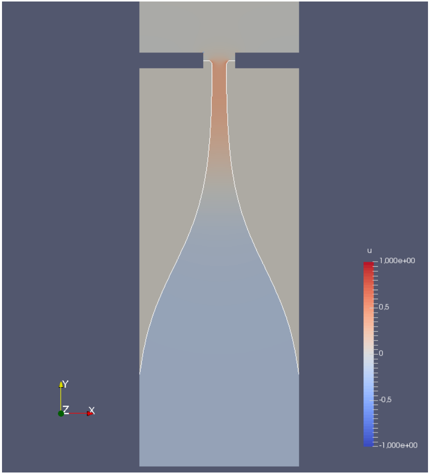
Resistance $Re(Z)$.

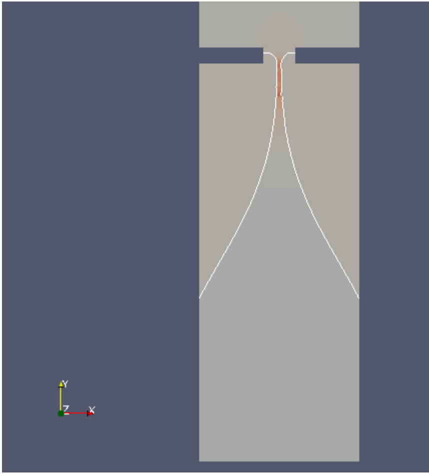


Absorption $\alpha = 1 - |R|^2$.

Results

Viscosity: $Z_T = 1.0$, $k = [0.15 : 0.25]$





Initial condition.
Coloured by $\nabla_{\delta\boldsymbol{\theta}}\mathcal{J}$.

Converged.

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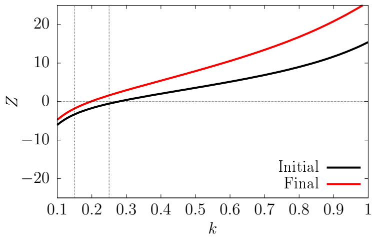

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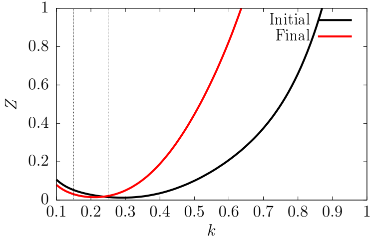

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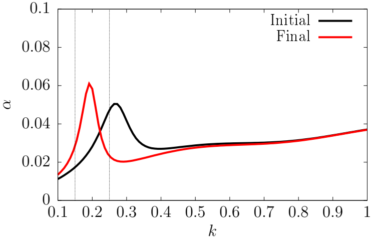

Absorption $\alpha = 1 - |R|^2$.

Conclusion

Summary:

- Shape optimisation for impedance matching.
- Viscous model: compromise between Helmholtz and full linearised Navier-Stokes.

Perspectives:

- Optimise from efficient/realistic configuration.
- Different cost functional (Penalty to initial guess, absorption, . . .).

Thank you for your attention.